

Topic Outline

3c3. Error Analysis

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Reporting Errors

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Accuracy and Precision

Accuracy

A measure of how close a result (or expected value) is to the “true” value.

Precision

A measure of how reproducibly we will obtain a result when the experiment is repeated under the same conditions.

Axioms

A result can be measured precisely yet still be inaccurate.

An imprecise result can be exact.

Definition

Random Errors

Random errors influence the result in a random (statistically unpredictable) manner.

All experimental systems will have some degree of random error!

This does not mean we have lost control of the cause-effect nature of the experiment!

Random errors effect the precision of a result, not its accuracy.

Treatment

Random Errors

The magnitude of random error in a result is determined by measuring the result repeatedly under the same experimental conditions.

Repeat your experiments!

Definition

Systematic Errors

Systematic errors cause the result to deviate from the true value in a systematic way.

Additive $x_M = x_T + C$

Multiplicative $x_M = C x_T$

Non-linear $x_M = f(x_T) x_T$

where x_M is the measured value, x_T is the true value, and C is a constant.

Systematic errors effect the accuracy of a result, not its precision.

Treatment

Systematic Errors

The magnitude of a systematic error in a result is determined by comparing the measured result against

- a theoretical predication

- a result measured in an independent lab

Compare your results with theory or literature values!

Definition

Measurement Errors

Measurement errors determine limit how well we can measure a result using the given experiment equipment.

**All experimental systems will
have some degree of
measurement error!**

Measurement errors effect the precision of a result, not its accuracy.

Treatment

Measurement Errors

The magnitude of measurement errors in a result can be reduced by using a more precise measuring system.

Know the limitations of your experimental equipment with regard to the variables in the experiment!

Reporting Errors

Absolute Errors $x \pm \Delta x$

Relative Errors $x \pm x\%$

They are related by

$$100 (\Delta x / x) = x\%$$

Significant Digits

Be aware of the limitations imposed on the number of significant digits in your result by the magnitude of the error.

Only one of these reported values for a composition uses the correct format.
Which one?

20.15 at %

20.2 ± 2 at %

20.2 ± 1.5 at %

20 ± 2 at %

Linear Error Analysis

Linear error analysis determines how errors propagate through a calculation.

Assumptions

errors are **uncorrelated** (they happen independently)

measured values are **normally distributed** (Gaussian)

Implications

variance of a value is determined by variances of events leading to that value

Adding Absolute Error

Add absolute errors
when values are
added or subtracted.

if

$$Y = \sum a_i x_i$$

then

$$(\Delta Y)^2 = \sum (a_i \Delta x_i)^2$$

Adding Relative Error

Add relative errors
when values are
multiplied or divided.

if

$$Y = \prod a_i x_i^{n_i}$$

then

$$(\Delta Y/Y)^2 = \sum (n_i \Delta x_i/x_i)^2$$